Übung3ANum

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9)

$L_2 L_1 A = R$ $\qquad -\frac{1}{2} + \frac{1}{4}$

A 9 a)

$$\left(\begin{array}{ccc|c} 6 & -4 & 7 & \frac{41}{12} \\ -12 & 5 & -12 & -\frac{23}{3} \\ 18 & 0 & 22 & \frac{29}{2} \end{array}\right) \cdot 2 \quad \cdot(-3)$$

$108 : 12 = 9$
90
18

$\frac{82}{12} - \frac{22}{3}$

$\frac{81}{12} - \frac{88}{12} = -\frac{6}{12} = -\frac{1}{2}$

$-\frac{123}{12} + \frac{29}{2}$

$\frac{-123 + 120 + 54}{12} = \frac{51}{12}$

$$\left(\begin{array}{ccc|c} 6 & -4 & 7 & \frac{41}{12} \\ 0 & -3 & 2 & -\frac{1}{2} \\ 0 & 12 & 1 & \frac{51}{12} \end{array}\right) \cdot 4 \qquad L_1 \begin{pmatrix} 1 & 0 & 0 \\ +\frac{1}{2} & 1 & 0 \\ -\frac{1}{3} & 0 & 1 \end{pmatrix}$$

$69 : 3 = 23$
6
09

$51 - 24 = 27$

$108 : 3 = 36$
9
18

$$\left(\begin{array}{ccc|c} 6 & -4 & 7 & \frac{41}{12} \\ 0 & -3 & 2 & -0{,}5 \\ 0 & 0 & 9 & \frac{27}{12} \end{array}\right) \qquad L_2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/4 & 1 \end{pmatrix}$$

R

$414 : 6 = 69$
36
54

54
140
189

1, $9x_3 = \frac{27}{12}$

$x_3 = \frac{27}{12 \cdot 9} = \frac{27}{108}$

$36 \cdot 4 = 120 + 24 = 144$

$9 \cdot 41 = 360 + 9 = 369$

2, $-3x_2 + 2x_3 = -0{,}5$

$\Rightarrow x_2 = \frac{2x_3 + 0{,}5}{3} = \frac{\frac{54}{108} + \frac{1}{2}}{3} = \frac{\frac{54}{108} + \frac{54}{108}}{3} = \frac{1}{3}$

3, $6x_1 - 4x_2 + 7x_3 = \frac{41}{12}$

$\Rightarrow x_1 = \left(4x_2 - 7x_3 + \frac{41}{12}\right) \cdot \frac{1}{6}$

$= \left(\frac{4}{3} - \frac{189}{108} + \frac{41}{12}\right) \frac{1}{6}$

$= \left(\frac{144}{108} - \frac{189}{108} + \frac{369}{108}\right) \cdot \frac{1}{6}$

$= \left(\frac{45}{108} + \frac{369}{108}\right) \cdot \frac{1}{6} = \frac{414}{108 \cdot 6} = \frac{69}{108}$

$$\left(\begin{array}{ccc|c} 6 & -4 & 7 & \frac{41}{72} \\ 0 & -3 & 2 & -\frac{45}{8} \\ 0 & 0 & 9 & \frac{4253}{288} \end{array}\right)$$

$$9x_3 = \frac{4253}{288}$$

$$x_3 = \frac{4253}{2592}$$

$$-3x_2 = -\frac{45}{8} + 2 \cdot \frac{4253}{2592}$$

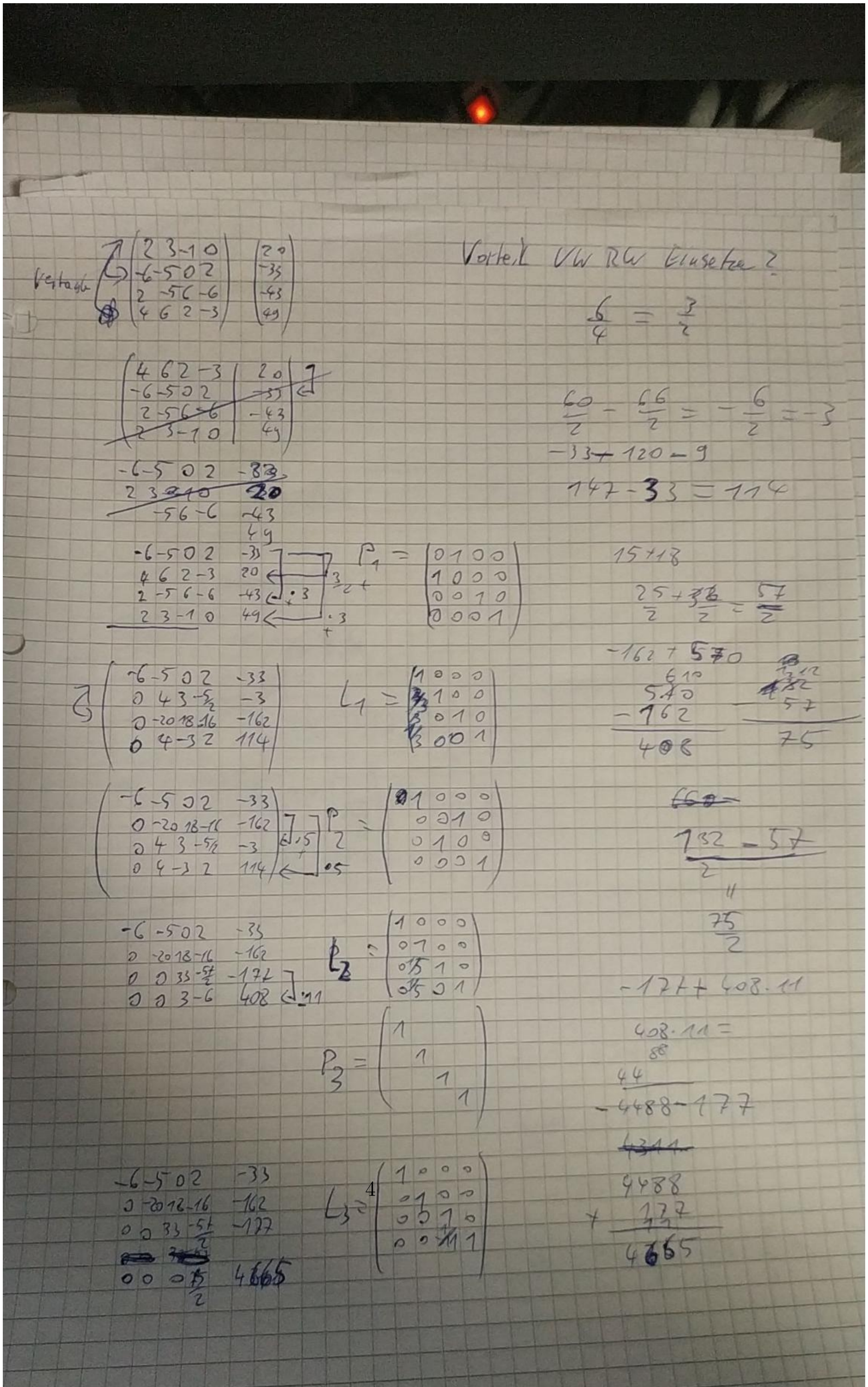
$$x_2 = -\frac{1}{3}\left(-\frac{45}{8} + 2 \cdot \frac{4253}{2592}\right)$$

$$x_2 = \frac{3037}{3888}$$

$$x_1 = \frac{1}{6}\left(4 \cdot \frac{3037}{3888} - 7 \cdot \frac{4253}{2592} + \frac{41}{72}\right)$$

$$x_1 = -\frac{38499}{46656}$$

11)


Vertausche
Vorteil VW RW Einsetzen ?
$P_1 =$
$L_1 =$
$P_2 =$
$L_2 =$
$P_3 =$
$L_3 =$
$-177 + 408 \cdot 11$

$$L = L_1^{-1} P_2 L_3^{-1}$$

$$= [illegible] L_1^{-1} P_2 P_3 P_3 L_2^{-1} P_3 P_4 P_3 P_3^{-1} P_4 [illegible]$$

$$= P_2 L_1^{-1} P_2 I L_2^{-1} I \cdot I L_3^{-1} \cdot I$$

$$= P_2 L_1^{-1} P_2 L_2^{-1} L_3^{-1}$$

$$= P_2 L_1^{-1} P_2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & [illegible] & 1 & 0 \\ 0 & -3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & [illegible] & 1 \end{pmatrix}$$

$$= P_2 L_1^{-1} P_2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & -3 & [illegible] & 1 \end{pmatrix}$$

$$= P_2 L_1^{-1} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & -3 & [illegible] & 1 \end{pmatrix}$$

$$= P_2 L_1^{-1} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & [illegible] & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ [illegible] & 0 & 1 & 0 \\ -3 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & [illegible] & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & [illegible] & 1 \end{pmatrix}$$

$$= \frac{23}{36}$$

$4 \cdot 2$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{5} & 1 & 0 \\ \frac{1}{3} & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{5} & \frac{1}{11} & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & 1 & 0 & 0 \\ \frac{2}{3} & \frac{1}{5} & 1 & 0 \\ \frac{1}{3} & \frac{1}{5} & \frac{1}{11} & 1 \end{pmatrix}$$

$$PA = LR \qquad PA = LR$$

$$\cancel{L^{-1} A' = R}$$

$$PAx = Pb$$

$$PAx = Pb$$

$$Pb = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 20 \\ -33 \\ -43 \\ 49 \end{pmatrix} = \begin{pmatrix} -33 \\ -43 \\ 20 \\ 49 \end{pmatrix}$$

$$L\underbrace{Rx}_{z} = Pb$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 & 0 \\ -\frac{2}{3} & -\frac{1}{5} & 1 & 0 \\ -\frac{1}{3} & -\frac{1}{5} & -\frac{1}{11} & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} -33 \\ -43 \\ 20 \\ 49 \end{pmatrix}$$

$$z_1 = -33$$

$$z_2 = -43 - \frac{1}{3} \cdot 33 = -54$$

$$z_3 = 20 + \frac{1}{5} \cdot -54 + \frac{2}{3} \cdot -33$$

$$= -\frac{64}{5}$$

$$z_4 = 49 + \frac{1}{3} \cdot -33 + \frac{1}{5} \cdot (-54) + \frac{1}{11} \cdot -\frac{64}{5}$$

$$= \frac{1432}{55}$$

36

2 3 4

$$= \frac{29}{2} - \frac{1}{3} \cdot \frac{41}{12} + \frac{1}{4} \cdot \frac{45}{8}$$

$$= \frac{29}{2} - \frac{41}{36} + \frac{45}{32} = \frac{522 - 41}{36}$$

$$Rx = \begin{pmatrix} -33 \\ -54 \\ -645 \\ 143255 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} -6 & -5 & 0 & 2 & -33 \\ 0 & -20 & 18 & -16 & -54 \\ 0 & 0 & 33 & -\frac{57}{2} & -645 \\ 0 & 0 & 0 & \frac{73}{2} & 143255 \end{array}\right)$$

$$x_4 = \frac{2864}{4125}$$

$$x_3 = \left(2\left(\frac{57}{2}\right)\cdot x_4 - \frac{64}{5}\right)\cdot\frac{1}{33} = \frac{9608}{65375}$$

$$x_2 = \left(-54 + 16x_4 - 18x_3\right)\cdot\frac{1}{-20} = \frac{211913}{90750}$$

$$x_1 = \frac{1}{-6}\left(-33 + 5\cdot\frac{211913}{90750} - 2\cdot\frac{2864}{4125}\right)$$

$$x_1 = \frac{687067}{181500}$$