

Übung3ANum

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May 2022

9)

$L_1 L_2 A = R$

Ag a)

$$\left(\begin{array}{ccc|ccc} 6 & -4 & 7 & 41 & 72 & 0 \\ -12 & 5 & -12 & -22 & 72 & 0 \\ 18 & 0 & 22 & 29 & 72 & 0 \end{array} \right) \begin{array}{l} \cdot 2 \\ \cdot 3 \\ \cdot 2 \end{array} \rightarrow \left(\begin{array}{ccc|ccc} 12 & -8 & 14 & 82 & 144 & 0 \\ -12 & 5 & -12 & -22 & 72 & 0 \\ 36 & 0 & 44 & 58 & 144 & 0 \end{array} \right) \begin{array}{l} - \\ + \\ + \end{array}$$

$$\left(\begin{array}{ccc|ccc} 12 & -8 & 14 & 82 & 144 & 0 \\ 0 & 5 & -12 & -22 & 72 & 0 \\ 0 & 0 & 30 & -24 & 72 & 0 \end{array} \right) \begin{array}{l} : 12 \\ : 5 \\ : 30 \end{array} \rightarrow \left(\begin{array}{ccc|ccc} 1 & -\frac{2}{3} & \frac{7}{6} & \frac{41}{6} & 12 & 0 \\ 0 & 1 & -\frac{4}{3} & -\frac{22}{5} & \frac{72}{5} & 0 \\ 0 & 0 & 1 & -\frac{2}{5} & \frac{24}{5} & 0 \end{array} \right) \begin{array}{l} \cdot 6 \\ \cdot 3 \\ \cdot 5 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & -\frac{2}{3} & \frac{7}{6} & \frac{41}{6} & 12 & 0 \\ 0 & 1 & -\frac{4}{3} & -\frac{22}{5} & \frac{72}{5} & 0 \\ 0 & 0 & 1 & -\frac{2}{5} & \frac{24}{5} & 0 \end{array} \right) \begin{array}{l} + \\ + \\ + \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 100 & 10 & 0 \\ 0 & 1 & 0 & 10 & 10 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array} \right) \begin{array}{l} : 10 \\ : 10 \\ : 10 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array} \right) \begin{array}{l} : 10 \\ : 10 \\ : 10 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right)$$

$108 : 12 = 9$
 90
 18

$82 - \frac{22}{3}$
 $\frac{81}{12} - \frac{88}{72} = -\frac{6}{12}$
 $= -\frac{1}{2}$
 $-\frac{123}{72} + \frac{29}{2}$
 $-\frac{123 + 720 + 54}{72} = \frac{51}{12}$

$69 : 3 = 23$
 $51 - 24 = 27$
 $108 : 3 = 36$
 54
 $414 : 6 = 69$
 140
 189
 $36 \cdot 4 = 120 + 24 = 144$
 $9 \cdot 41 = 360 + 9 = 369$

108
 23
 36

$1. \quad 9x_3 = \frac{27}{12}$
 $x_3 = \frac{27}{12 \cdot 9} = \frac{27}{108}$

$2. \quad -3x_2 + 2x_3 = -0,5$
 $\Rightarrow x_2 = \frac{2x_3 + 0,5}{3} = \frac{\frac{54}{108} + \frac{1}{2}}{3} = \frac{\frac{54}{108} + \frac{54}{108}}{3} = \frac{1}{3}$

$3. \quad 6x_1 - 4x_2 + 7x_3 = \frac{41}{12}$
 $\Rightarrow x_1 = \left(\frac{4x_2 - 7x_3 + \frac{41}{12}}{6} \right) \cdot \frac{1}{6}$
 $= \left(\frac{4}{3} - \frac{189}{108} + \frac{41}{12} \right) \cdot \frac{1}{6}$
 $= \left(\frac{144}{108} - \frac{189}{108} + \frac{369}{108} \right) \cdot \frac{1}{6}$
 $= \left(\frac{45}{108} + \frac{369}{108} \right) \cdot \frac{1}{6} = \frac{414}{108 \cdot 6} = \frac{369}{108}$

$$\begin{array}{ccc|c} 6 & -4 & 7 & \frac{41}{72} \\ 0 & -3 & 2 & -\frac{45}{8} \\ 0 & 0 & 9 & \frac{4253}{288} \end{array}$$

$$9X_3 = \frac{4253}{288}$$

$$X_3 = \frac{4253}{2592}$$

$$-3X_2 = -\frac{45}{8} + 2 \cdot \frac{4253}{2592}$$

$$X_2 = -\frac{1}{3} \left(-\frac{45}{8} + 2 \cdot \frac{4253}{2592} \right)$$

$$X_2 = \frac{3037}{3888}$$

$$X_1 = \frac{1}{6} \left(4 \cdot \frac{3037}{3888} - 7 \cdot \frac{4253}{2592} + \frac{41}{72} \right)$$

$$X_1 = \frac{38499}{46656}$$

11)

Vorteil VW RW einsetzen?

$$\frac{6}{4} = \frac{3}{2}$$

$$\frac{60}{2} - \frac{66}{2} = -\frac{6}{2} = -3$$

$$-33 + 120 - 9$$

$$147 - 33 = 114$$

$$15 + 18$$

$$\frac{25 + 36}{2} = \frac{57}{2}$$

$$-162 + 570$$

$$\frac{610}{2} = 305$$

$$540$$

$$-162$$

$$408 \quad 75$$

$$\frac{752 - 57}{2}$$

$$\frac{75}{2}$$

$$-177 + 408 \cdot 11$$

$$\frac{408 \cdot 11}{88} = 44$$

$$-4488 - 177$$

$$4344$$

$$4488$$

$$+ 177$$

$$4665$$

Handwritten work on grid paper showing a linear programming problem. The problem is to maximize profit (Z) given constraints on resources (A, B, C, D). The constraints are:

$$\begin{aligned} 2x_1 + 3x_2 &\leq 20 \\ 6x_1 + 5x_2 &\leq 33 \\ 2x_1 + 5x_2 &\leq 43 \\ 4x_1 + 6x_2 &\leq 49 \end{aligned}$$

The objective function is:

$$Z = 6x_1 + 4x_2$$

The feasible region is determined by the intersection of these constraints. The vertices of the feasible region are found by solving the system of equations:

$$\begin{aligned} 2x_1 + 3x_2 &= 20 \\ 6x_1 + 5x_2 &= 33 \end{aligned}$$

The optimal solution is found at the vertex (3, 2), where the profit is:

$$Z = 6(3) + 4(2) = 18 + 8 = 26$$

The handwritten work shows the steps of the simplex method, including the initial tableau, the selection of the pivot element, and the resulting tableaux. The final tableau shows the optimal solution at (3, 2) with a profit of 26.

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

or

Case 2: $Q=0$

$$L = L_1^{-1} L_2^{-1} L_3^{-1}$$

$$= P_1^{-1} L_1^{-1} P_1 P_2 P_3 P_2^{-1} L_2^{-1} P_2^{-1} P_3^{-1} P_3^{-1} P_4^{-1} P_4^{-1}$$

$$= P_1^{-1} L_1^{-1} P_1 I L_2^{-1} I P_2^{-1} L_3^{-1} I$$

$$= P_1^{-1} L_1^{-1} P_1 L_2^{-1} L_3^{-1}$$

~~$$= P_1^{-1} L_1^{-1} P_1 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$~~

$$= P_2 L_1^{-1} P_2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= P_2 L_1^{-1} P_2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= P_2 L_1^{-1} P_2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= P_2 L_1^{-1} P_2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \frac{23}{36}$$

4.2

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$PA = LR$$

$$PA = CR$$

~~$$PA = R$$~~

$$PA = b$$

$$Ax = Pb$$

$$Pb = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 20 \\ -33 \\ -43 \\ 49 \end{pmatrix} = \begin{pmatrix} -33 \\ 43 \\ 20 \\ 49 \end{pmatrix}$$

$$L \cdot Rx = Pb$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} -33 \\ 43 \\ 20 \\ 49 \end{pmatrix}$$

$$z_1 = -33$$

$$z_2 = -43 + \frac{1}{5} \cdot 33 = -54$$

$$z_3 = 20 + \frac{1}{5} \cdot (-54) + \frac{2}{3} \cdot (-33) = -64$$

$$z_4 = 49 + \frac{1}{3} \cdot (-33) + \frac{1}{5} \cdot (-54) + \frac{1}{11} \cdot (-6) = \frac{1432}{55}$$

36

6

$$= \frac{29}{2} - \frac{1}{3} \cdot \frac{41}{12} + \frac{1}{4} \cdot \frac{45}{8}$$

$$= \frac{29}{2} - \frac{41}{36} + \frac{45}{32} = \frac{522}{36} - \frac{41}{36}$$

$$R \cdot X = \begin{pmatrix} -33 \\ -54 \\ -645 \\ 112055 \end{pmatrix}$$

$$\begin{array}{l|l} -6.502 & -33 \\ 0.201816 & -54 \\ 0.03152 & -645 \\ 0.0072 & 112055 \end{array}$$

$$X_4 = \frac{2864}{6425}$$

$$X_3 = \left(2 \cdot \frac{57}{2} \cdot X_4 - \frac{54}{5} \right) \cdot \frac{1}{33} = \frac{9608}{65375}$$

$$X_2 = \left(-54 - 16 \cdot X_4 - 18 \cdot X_3 \right) \cdot \frac{1}{-20} = \frac{211913}{90750}$$

$$X_1 = \frac{1}{-6} \left(-33 - 5 \cdot \frac{211913}{90750} - 2 \cdot \frac{2864}{6425} \right)$$

$$X_1 = \frac{687067}{131500}$$